

5 5 practice inequalities involving absolute value

5 5 practice inequalities involving absolute value are essential concepts in algebra that help students understand how to manipulate and solve equations involving the absolute value function. Absolute value represents the distance of a number from zero on the number line, regardless of direction. This characteristic leads to unique properties when dealing with inequalities, as they can yield multiple solutions. In this article, we will explore five practice inequalities involving absolute values, providing detailed explanations and step-by-step solutions to reinforce your understanding.

Understanding Absolute Value

Before diving into practice inequalities, it's crucial to understand what absolute value is. The absolute value of a real number x , denoted as $|x|$, is defined as follows:

- If $x \geq 0$, then $|x| = x$
- If $x < 0$, then $|x| = -x$

This definition implies that absolute values are always non-negative. The absolute value can be visualized as the distance from zero on the number line, which is always a positive quantity.

Why Solve Inequalities with Absolute Values?

Inequalities involving absolute values can represent a range of values rather than a single solution. Solving these inequalities can help in various real-world applications, such as:

- Measurement tolerances
- Error margins in calculations
- Optimization problems in economics and engineering

Understanding how to solve these inequalities is a foundational skill in algebra, preparing students for more advanced mathematics.

Practice Inequality 1: Simple Absolute Value Inequality

Inequality:

$$|x - 3| < 5$$

Solution:

To solve the inequality $|x - 3| < 5$, we need to break it into two separate inequalities based on the definition of absolute value:

- $x - 3 < 5$
- $x - 3 > -5$

Now, we will solve each inequality separately:

- For $x - 3 < 5$:

$$\begin{aligned} &| \\ &x < 8 \\ &| \end{aligned}$$

- For $x - 3 > -5$:

$$\begin{aligned} &| \\ &x > -2 \\ &| \end{aligned}$$

Combining these results, we find:

$$\begin{aligned} &| \\ &-2 < x < 8 \\ &| \end{aligned}$$

Thus, the solution to the inequality $|x - 3| < 5$ is $(-2, 8)$.

Practice Inequality 2: Absolute Value Greater Than

Inequality:

$$\begin{aligned} &| \\ &|2x + 1| \geq 3 \\ &| \end{aligned}$$

Solution:

Similar to the first problem, we will break this absolute value inequality into two cases:

- $2x + 1 \geq 3$
- $2x + 1 \leq -3$

Now, solve each case:

1. For $(2x + 1 \geq 3)$:

$$2x \geq 2 \quad \Rightarrow \quad x \geq 1$$

2. For $(2x + 1 \leq -3)$:

$$2x \leq -4 \quad \Rightarrow \quad x \leq -2$$

Combining these results gives us:

$$x \leq -2 \quad \text{or} \quad x \geq 1$$

Thus, the solution to the inequality $(|2x + 1| \geq 3)$ is $((-\infty, -2] \cup [1, \infty))$.

Practice Inequality 3: Compound Absolute Value Inequality

Inequality:

$$|x + 4| < 2$$

Solution:

We will again consider the two cases for this inequality:

- $(x + 4 < 2)$
- $(x + 4 > -2)$

Now, solve each case:

1. For $(x + 4 < 2)$:

$$x < -2$$

2. For $(x + 4 > -2)$:

$$x > -6$$

Combining these results gives us:

$$\begin{aligned} & \backslash \\ & -6 < x < -2 \\ & \backslash \end{aligned}$$

Thus, the solution to the inequality $|x + 4| < 2$ is $(-6, -2)$.

Practice Inequality 4: Absolute Value with a Quadratic Expression

Inequality:

$$\begin{aligned} & \backslash \\ & |x^2 - 4| \leq 3 \\ & \backslash \end{aligned}$$

Solution:

To solve this inequality, we need to consider two cases:

- $|x^2 - 4| \leq 3$
- $|x^2 - 4| \geq -3$

Now, let's solve these inequalities:

- For $|x^2 - 4| \leq 3$:

$$\begin{aligned} & \backslash \\ & x^2 \leq 7 \quad \Rightarrow \quad -\sqrt{7} \leq x \leq \sqrt{7} \\ & \backslash \end{aligned}$$

- For $|x^2 - 4| \geq -3$:

$$\begin{aligned} & \backslash \\ & x^2 \geq 1 \quad \Rightarrow \quad x \leq -1 \quad \text{or} \quad x \geq 1 \\ & \backslash \end{aligned}$$

Combining these results leads us to find the intersection of the two conditions:

- From $-\sqrt{7} \leq x \leq \sqrt{7}$
- From $x \leq -1$ or $x \geq 1$

Thus, the solution is:

$$\begin{aligned} & \backslash \\ & (-\sqrt{7}, -1] \cup [1, \sqrt{7}) \\ & \backslash \end{aligned}$$

Practice Inequality 5: Absolute Value with a Linear Expression

Inequality:

$$\backslash$$
$$|3x - 5| > 4$$
$$\backslash$$

Solution:

This inequality can also be broken down into two parts:

1. $\backslash (3x - 5 > 4 \backslash$
2. $\backslash (3x - 5 < -4 \backslash$

Now, let's solve each case:

1. For $\backslash (3x - 5 > 4 \backslash$:

$$\backslash$$
$$3x > 9 \quad \backslash \text{Rightarrow} \quad \backslash \text{quad} \quad x > 3$$
$$\backslash$$

2. For $\backslash (3x - 5 < -4 \backslash$:

$$\backslash$$
$$3x < 1 \quad \backslash \text{Rightarrow} \quad \backslash \text{quad} \quad x < \frac{1}{3}$$
$$\backslash$$

Combining these results gives us:

$$\backslash$$
$$x < \frac{1}{3} \quad \backslash \text{quad} \quad \backslash \text{text{or}} \quad \backslash \text{quad} \quad x > 3$$
$$\backslash$$

Thus, the solution to the inequality $\backslash (|3x - 5| > 4 \backslash$ is $\backslash ((-\infty, \frac{1}{3}) \cup (3, \infty) \backslash$.

Conclusion

In this article, we explored five practice inequalities involving absolute values, showcasing the methods to solve and interpret each case. These inequalities demonstrate how absolute values can lead to a range of solutions, rather than a single answer. Mastering these skills is crucial for success in algebra and serves as a stepping stone for more advanced mathematical concepts. Regular practice with such inequalities will enhance problem-solving abilities and deepen your understanding of algebraic principles.

Frequently Asked Questions

What is an absolute value inequality?

An absolute value inequality is an inequality that contains an absolute value expression. It typically takes the form $|x| < a$ or $|x| > b$, where a and b are constants.

How do you solve the inequality $|x| < 5$?

To solve $|x| < 5$, you split it into two inequalities: $-5 < x < 5$. This means that x can take any value between -5 and 5 , not including the endpoints.

What does the inequality $|x| > 5$ represent?

The inequality $|x| > 5$ means that x is either less than -5 or greater than 5 . Therefore, the solution is $x < -5$ or $x > 5$.

How do you graph the solution to $|x| < 3$?

To graph $|x| < 3$, you would draw a number line and shade the region between -3 and 3 , excluding the endpoints, since the inequality is strict.

What are the steps to solve $|2x - 1| \leq 4$?

First, split the inequality into two cases: $2x - 1 \leq 4$ and $2x - 1 \geq -4$. Then solve each case for x . For the first case, you get $x \leq 2.5$, and for the second case, $x \geq -1.5$. Therefore, the solution is $-1.5 \leq x \leq 2.5$.

Can absolute value inequalities have no solution?

Yes, absolute value inequalities can have no solution. For example, the inequality $|x| < -1$ has no solution because absolute values are always non-negative, and they cannot be less than a negative number.

What is the difference between strict and non-strict absolute value inequalities?

Strict inequalities (like $|x| < a$) do not include the endpoints, while non-strict inequalities (like $|x| \leq a$) do include the endpoints. This affects how you express the solution set.

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