arithmetic sequences answer key

Arithmetic sequences answer key refers to the solutions and explanations related to problems involving arithmetic sequences, a fundamental concept in mathematics. An arithmetic sequence is a sequence of numbers in which the difference between consecutive terms is constant. This constant difference is known as the "common difference." Understanding arithmetic sequences is crucial for students as they form the foundation for more advanced mathematical concepts, including series, functions, and algebra. This article aims to provide a comprehensive overview of arithmetic sequences, their properties, and common problems, along with their solutions and an answer key.

Understanding Arithmetic Sequences

Definition

An arithmetic sequence is defined by the formula:

$$ a_n = a_1 + (n - 1) \cdot d $$

Where:
- $ a_n $ = n-th term of the sequence
- $ a_1 $ = first term of the sequence
- $ d $ = common difference
- $ n $ = number of terms

In this formula, each term is generated by adding the common difference to the previous term.

Examples of Arithmetic Sequences

1. Example 1: 2, 5, 8, 11, 14, ...

- Here, the first term $a_1 = 2$ and the common difference $d = 3$.

2. Example 2: 10, 7, 4, 1, -2, ...

- In this sequence, the first term $a_1 = 10$ and the common difference $d = -3$.

3. Example 3: 1, 1.5, 2, 2.5, 3, ...

- For this sequence, the first term $a_1 = 1$ and the common difference $d = 0.5$.

Properties of Arithmetic Sequences

1. Common Difference

The common difference d can be calculated by subtracting any term from the subsequent term.

For example, in the sequence 4, 7, 10, the common difference is $7 - 4 = 3$ or $10 - 7 = 3$.

2. Sum of the First n Terms

The sum S_n of the first n terms of an arithmetic sequence can be calculated using the formula:

$$S_n = \frac{n}{2} \cdot (a_1 + a_n)$$

Alternatively, this can also be expressed as:

$$S_n = \frac{n}{2} \cdot (2a_1 + (n - 1)d)$$

This formula helps in quickly calculating the sum without needing to list all the terms.

3. Finding the n-th Term

To find the n-th term of an arithmetic sequence, one can use the previously mentioned formula for a_n.

4. Infinite Arithmetic Sequences

While arithmetic sequences can theoretically be infinite, they do not converge to a limit like geometric sequences. The terms will continue indefinitely, either increasing or decreasing based on the sign of the common difference.

Common Problems Involving Arithmetic Sequences

Problem 1: Finding the n-th Term

Given an arithmetic sequence where the first term is 5 and the common difference is 4, find the 10th term.

Solution:
Using the formula for the n-th term:

$$a_n = a_1 + (n - 1) \cdot d$$
$$a_{10} = 5 + (10 - 1) \cdot 4$$
$$a_{10} = 5 + 36 = 41$$

Problem 2: Finding the Common Difference

If the first term of an arithmetic sequence is 12, and the fifth term is 36, find the common difference.

Solution:

Using the formula for the n-th term:

$$ a_n = a_1 + (n - 1) \cdot d $$

$$ a_5 = 12 + (5 - 1) \cdot d $$

$$ 36 = 12 + 4d $$

$$ 24 = 4d $$

$$ d = 6 $$

Problem 3: Sum of the First n Terms

Calculate the sum of the first 20 terms of an arithmetic sequence where the first term is 3 and the common difference is 2.

Solution:

First, find the 20th term:

$$ a_{20} = 3 + (20 - 1) \cdot 2 $$

$$ a_{20} = 3 + 38 = 41 $$

Now, use the sum formula:

$$ S_{20} = \frac{20}{2} \cdot (3 + 41) $$

$$ S_{20} = 10 \cdot 44 = 440 $$

Problem 4: Finding the Total Number of Terms

An arithmetic sequence has a first term of 8 and a last term of 50, with a common difference of 2. How many terms are in this sequence?

Solution:

Using the formula for the n-th term:

$$ a_n = a_1 + (n - 1) \cdot d $$

Set $ a_n = 50 $:

$$ 50 = 8 + (n - 1) \cdot 2 $$

$$ 50 - 8 = (n - 1) \cdot 2 $$

$$ 42 = (n - 1) \cdot 2 $$

$$ n - 1 = 21 $$

$$ n = 22 $$

Answer Key for Common Problems

1. Finding the n-th Term:

- Problem: 10th term of an arithmetic sequence with first term 5 and common difference 4.

- Answer: 41

2. Finding the Common Difference:

- Problem: First term 12, fifth term 36.

- Answer: 6

3. Sum of the First n Terms:

- Problem: Sum of first 20 terms with first term 3 and common difference 2.

- Answer: 440

4. Finding the Total Number of Terms:

- Problem: First term 8, last term 50, common difference 2.

- Answer: 22

Conclusion

Arithmetic sequences are a critical concept in mathematics, providing a framework for understanding linear relationships and series. The ability to identify the terms and properties of arithmetic sequences is essential for students as they progress in their mathematical education. Through various problems and their corresponding solutions, learners can gain a solid grasp of how to work with arithmetic sequences effectively. The answer key provided serves as a valuable resource for verifying understanding and mastering this fundamental topic.

Frequently Asked Questions

What is an arithmetic sequence?

An arithmetic sequence is a sequence of numbers in which the difference between consecutive terms is constant.

How do you find the nth term of an arithmetic sequence?

The nth term of an arithmetic sequence can be found using the formula: $a_n = a_1 + (n - 1)d$, where a_1 is the first term and d is the common difference.

What is the common difference in an arithmetic sequence?

The common difference is the fixed amount added to each term to get the next term in the sequence.

How can you determine if a sequence is arithmetic?

To determine if a sequence is arithmetic, check if the difference between consecutive terms is the same throughout the sequence.

What is the formula for the sum of the first n terms of an arithmetic sequence?

The sum of the first n terms (S_n) of an arithmetic sequence can be calculated using the formula: $S_n = n/2 (a_1 + a_n)$, or $S_n = n/2 (2a_1 + (n - 1)d)$.

Can an arithmetic sequence have negative common differences?

Yes, an arithmetic sequence can have negative common differences, which means the terms will decrease as you progress in the sequence.

What is the significance of the first term in an arithmetic sequence?

The first term is crucial as it serves as the starting point for the sequence, determining all subsequent terms based on the common difference.

Arithmetic Sequences Answer Key