college algebra problems and answers

College algebra problems and answers are essential components of the curriculum for many college students, providing them with the foundational skills needed for higher-level mathematics and various applications in science, engineering, and economics. This article will explore some common types of college algebra problems, provide detailed solutions, and offer strategies for mastering these concepts. We will cover topics such as equations, inequalities, functions, and polynomials, ensuring a comprehensive understanding of the subject.

Understanding Equations

Equations are fundamental to algebra, as they express the relationship between different quantities. Solving equations involves finding the value of the variable that makes the equation true.

Linear Equations

A linear equation is an equation of the first degree, meaning it has no exponents greater than one. The general form is:

$$ ax + b = 0 $$

Example Problem:

Solve the following linear equation:

$$ 3x + 5 = 14 $$

Solution:

1. Subtract 5 from both sides:
$$ 3x = 14 - 5 $$
$$ 3x = 9 $$

2. Divide both sides by 3:
$$ x = \frac{9}{3} $$
$$ x = 3 $$

Answer: $x = 3$

Quadratic Equations

Quadratic equations are polynomial equations of the second degree, usually in the form:

$$ax^2 + bx + c = 0$$

Example Problem:

Solve the quadratic equation:

$$x^2 - 4x - 5 = 0$$

Solution:

1. Factor the quadratic:
$$(x - 5)(x + 1) = 0$$

2. Set each factor to zero:
$$x - 5 = 0 \quad \Rightarrow \quad x = 5$$
$$x + 1 = 0 \quad \Rightarrow \quad x = -1$$

Answer: $x = 5$ or $x = -1$

Inequalities

Inequalities express a relationship where one quantity is greater than or less than another. Solving inequalities requires similar techniques as solving equations, but we must pay attention to the direction of the inequality sign.

Linear Inequalities

The general form of a linear inequality is:

$$ax + b < c$$

Example Problem:

Solve the following inequality:

$$2x - 3 > 5$$

Solution:

1. Add 3 to both sides:
$$2x > 5 + 3$$
$$2x > 8$$

2. Divide both sides by 2:
$$x > 4$$

Answer: $x > 4$

Compound Inequalities

Compound inequalities involve two inequalities that are combined into one statement.

Example Problem:

Solve the compound inequality:

$$ -2 < 3x + 1 \leq 8 $$

Solution:

1. Break it into two parts:
- First part: $-2 < 3x + 1$
- Second part: $3x + 1 \leq 8$

2. Solve the first part:
- Subtract 1:
$$ -3 < 3x $$
- Divide by 3:
$$ -1 < x $$

3. Solve the second part:
- Subtract 1:
$$ 3x \leq 7 $$
- Divide by 3:
$$ x \leq \frac{7}{3} $$

Combining Results:
$$ -1 < x \leq \frac{7}{3} $$

Answer: $-1 < x \leq \frac{7}{3}$

Functions

Functions are a critical concept in algebra, defined as a relation between a set of inputs and a set of possible outputs.

Evaluating Functions

To evaluate a function, substitute the input value into the function's formula.

Example Problem:

If $f(x) = 2x^2 + 3x - 5$, find $f(2)$.

Solution:

1. Substitute 2 for x:
$$f(2) = 2(2)^2 + 3(2) - 5$$
$$= 2(4) + 6 - 5$$
$$= 8 + 6 - 5$$
$$= 9$$

Answer: $f(2) = 9$

Finding the Domain and Range of Functions

The domain is the set of all possible input values (x-values), while the range is the set of all possible output values (y-values).

Example Problem:

Determine the domain and range of the function:

$$g(x) = \sqrt{x - 1}$$

Solution:

- Domain: The expression under the square root must be non-negative:
$$x - 1 \geq 0$$
$$x \geq 1$$
Thus, the domain is $[1, \infty)$.

- Range: The output of $g(x)$ is non-negative:
$$g(x) \geq 0$$
Thus, the range is $[0, \infty)$.

Answer: Domain: $[1, \infty)$; Range: $[0, \infty)$

Polynomials

Polynomials are expressions that involve variables raised to whole-number powers and their coefficients.

Adding and Subtracting Polynomials

To add or subtract polynomials, combine like terms.

Example Problem:

Add the following polynomials:

$$ (3x^2 + 2x - 5) + (4x^2 - 3x + 7) $$

Solution:

1. Combine like terms:
- $3x^2 + 4x^2 = 7x^2$
- $2x - 3x = -x$
- $-5 + 7 = 2$

The result is:
$$ 7x^2 - x + 2 $$

Answer: $7x^2 - x + 2$

Factoring Polynomials

Factoring polynomials involves expressing them as a product of their factors.

Example Problem:

Factor the polynomial:

$$ x^2 - 9 $$

Solution:

This is a difference of squares:
$$ x^2 - 9 = (x - 3)(x + 3) $$

Answer: $(x - 3)(x + 3)$

Conclusion

Mastering college algebra requires a solid understanding of various concepts, including equations, inequalities, functions, and polynomials. By practicing problems and applying the solutions provided in this article, students can develop the skills necessary to tackle more advanced mathematics. This foundational knowledge is not only crucial for academic success but also for real-world applications in diverse fields such as engineering, economics, and the sciences. With dedication and practice, anyone can excel in college algebra and beyond.

Frequently Asked Questions

What is the quadratic formula used to solve $ax^2 + bx + c = 0$?

The quadratic formula is $x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$.

How do you find the vertex of a parabola given in the form $y = ax^2 + bx + c$?

The vertex can be found using the formula $(-b/2a, f(-b/2a))$, where $f(x)$ is the original quadratic function.

What is the difference between a rational and an irrational number?

A rational number can be expressed as a fraction of two integers, while an irrational number cannot be expressed as a simple fraction.

How do you simplify the expression $(3x^2y)(2xy^3)$?

The simplified expression is $6x^3y^4$.

What are the steps to solve a system of equations using substitution?

1. Solve one equation for one variable. 2. Substitute that expression into the other equation. 3. Solve for the remaining variable. 4. Substitute back to find the first variable.

How do you determine if a function is even, odd, or neither?

A function $f(x)$ is even if $f(-x) = f(x)$, odd if $f(-x) = -f(x)$, and neither if it does not satisfy either condition.

What is the process for factoring a polynomial like $x^2 + 5x + 6$?

To factor, find two numbers that multiply to 6 (the constant term) and add to 5 (the coefficient of x). The factors are $(x + 2)(x + 3)$.

What does it mean for a function to be one-to-one?

A function is one-to-one if it assigns a unique output for each input, meaning no two different inputs produce the same output.

How can you find the x-intercepts of a function?

To find the x-intercepts, set the function equal to zero and solve for x.

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