DIFFERENTIAL CALCULUS PROBLEMS WITH SOLUTIONS

Differential calculus problems with solutions are fundamental to understanding the field of calculus. This branch of mathematics focuses on the concept of the derivative, which represents the rate at which a function is changing at any given point. By mastering differential calculus, students can analyze various types of functions and solve real-world problems involving motion, optimization, and more. In this article, we will explore some common differential calculus problems, provide step-by-step solutions, and highlight key concepts along the way.

Understanding the Derivative

Before diving into specific problems, it's important to understand what a derivative is. The derivative of a function $f(x)$ at a point $x = a$ is defined as:

$$
f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}
$$

This limit expresses the instantaneous rate of change of the function at the point a. The derivative can be interpreted as the slope of the tangent line to the curve defined by the function at that point.

Basic Derivative Problems

Let's start with some basic problems that involve finding derivatives using standard rules.

Problem 1: Find the derivative of $f(x) = 3x^2 + 5x - 4$

Solution:

To find the derivative $f'(x)$, we use the power rule, which states that the derivative of x^n is nx^{n-1}.

1. Differentiate $3x^2$:
$$
\frac{d}{dx}(3x^2) = 2 \cdot 3x^{2-1} = 6x
$$

2. Differentiate $5x$:
$$
\frac{d}{dx}(5x) = 5
$$

3. The derivative of a constant (-4) is 0.

Putting it all together, we have:
$$
f'(x) = 6x + 5
$$

PROBLEM 2: FIND THE DERIVATIVE OF \(g(x) = \sin(x) + \cos(x) \)

SOLUTION:

USING THE DERIVATIVES OF SINE AND COSINE, WE KNOW:
- \(\frac{d}{dx}(\sin(x)) = \cos(x) \)
- \(\frac{d}{dx}(\cos(x)) = -\sin(x) \)

THUS,
\[
g'(x) = \cos(x) - \sin(x)
\]

APPLYING THE PRODUCT AND QUOTIENT RULES

IN MORE COMPLEX PROBLEMS, WE OFTEN NEED TO APPLY THE PRODUCT RULE OR THE QUOTIENT RULE.

PROBLEM 3: FIND THE DERIVATIVE OF \(h(x) = x^2 \cdot \ln(x) \)

SOLUTION:

USING THE PRODUCT RULE, WHICH STATES THAT \((uv)' = u'v + uv' \):

LET \(u = x^2 \) AND \(v = \ln(x) \).

1. DIFFERENTIATE \(u \):
\[
u' = 2x
\]

2. DIFFERENTIATE \(v \):
\[
v' = \frac{1}{x}
\]

NOW APPLY THE PRODUCT RULE:
\[
h'(x) = u'v + uv' = (2x)(\ln(x)) + (x^2)\left(\frac{1}{x}\right)
\]
\[
h'(x) = 2x \ln(x) + x
\]

PROBLEM 4: FIND THE DERIVATIVE OF \(k(x) = \frac{x^3 - 1}{x + 1} \)

SOLUTION:

USING THE QUOTIENT RULE, WHICH STATES THAT \(\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \):

LET \(u = x^3 - 1 \) AND \(v = x + 1 \).

1. DIFFERENTIATE \(u \):

$$
u' = 3x^2
$$

2. Differentiate (v):
$$
v' = 1
$$

Now apply the quotient rule:
$$
k'(x) = \frac{(3x^2)(x + 1) - (x^3 - 1)(1)}{(x + 1)^2}
$$
$$
= \frac{3x^3 + 3x^2 - x^3 + 1}{(x + 1)^2}
$$
$$
= \frac{2x^3 + 3x^2 + 1}{(x + 1)^2}
$$

Higher-Order Derivatives

Calculus also allows us to find higher-order derivatives, which are derivatives of derivatives.

Problem 5: Find the second derivative of $(f(x) = 4x^3 - 3x^2 + 2)$

Solution:

First, we find the first derivative:
$$
f'(x) = 12x^2 - 6x
$$

Next, we find the second derivative:
$$
f''(x) = \frac{d}{dx}(12x^2 - 6x) = 24x - 6
$$

Applications of Differential Calculus

Differential calculus is not just an academic exercise; it finds applications in various fields such as physics, economics, and engineering. Here are a few examples:

1. Finding Maximum and Minimum Values: By setting the first derivative to zero, we can find critical points and determine whether they are maxima, minima, or points of inflection.

2. Analyzing Motion: The derivative of the position function $(s(t))$ gives us the velocity $(v(t))$, and the derivative of the velocity function gives us acceleration $(a(t))$.

3. Optimizing Functions: Businesses often use calculus to find the optimal production levels or pricing strategies that maximize profit or minimize costs.

Conclusion

In conclusion, differential calculus provides powerful tools for analyzing and understanding the behavior of functions. Through the problems presented in this article, we've seen how to compute derivatives using basic rules, apply product and quotient rules, and explore higher-order derivatives. Mastery of these concepts not only enhances mathematical skills but also prepares students for real-world applications in various domains. As you practice these problems, remember that understanding the principles behind the techniques is just as important as being able to perform the calculations.

Frequently Asked Questions

What is the derivative of the function $f(x) = 3x^2 + 2x - 5$?

The derivative $f'(x) = 6x + 2$.

How do you find the critical points of the function $f(x) = x^3 - 3x^2 + 4$?

To find the critical points, first compute the derivative $f'(x) = 3x^2 - 6x$. Set $f'(x) = 0$ to find critical points: $3x(x - 2) = 0$, giving critical points at $x = 0$ and $x = 2$.

What is the second derivative test and how is it applied?

The second derivative test involves finding the second derivative of a function to determine the concavity at critical points. If $f''(x) > 0$, the function has a local minimum; if $f''(x) < 0$, it has a local maximum.

How do you solve the optimization problem of maximizing the area of a rectangle with a fixed perimeter?

Let the length be x and width be y. Given the perimeter $P = 2(x + y)$, we have $y = P/2 - x$. The area $A = xy = x(P/2 - x)$. To maximize A, take the derivative $dA/dx = P/2 - 2x$, set it to zero to find $x = P/4$, giving maximum area when the rectangle is a square.

What is implicit differentiation and when is it used?

Implicit differentiation is used when dealing with functions defined implicitly rather than explicitly. For example, for the equation $x^2 + y^2 = 1$, differentiate both sides with respect to x to find $dy/dx = -x/y$.

What is the derivative of $\sin(x)$ and how is it applied in problems?

The derivative of $\sin(x)$ is $\cos(x)$. This is used in problems involving rates of change in trigonometric contexts, such as finding the slope of a sine curve at a given point.

How do you apply the chain rule to find the derivative of $f(g(x))$?

To apply the chain rule, use the formula $(f(g(x)))' = f'(g(x)) \, g'(x)$. For example, if $f(x) = x^2$ and $g(x) = \sin(x)$, then the derivative is $2\sin(x) \cos(x)$.

What is a tangent line and how do you find its equation?

A tangent line to a function at a point $(a, f(a))$ has the equation $y - f(a) = f'(a)(x - a)$. To find it, calculate $f'(a)$ and substitute into the equation.

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